τ magnetic moment in a $\gamma\gamma$ collider

L. Tabares and O. A. Sampayo
Departamento de Física, Universidad Nacional de Mar del Plata Funes 3350, (7600) Mar del Plata, Argentina
(Received 23 July 2001; published 6 February 2002)

We analyze different observables to study the magnetic dipole moment of the tau pairs produced by photon linear colliders. We use the circular polarized photon beam and study distributions of polarized τ final pairs to define sensible asymmetries to the magnetic dipole moment.

DOI: 10.1103/PhysRevD.65.053012 PACS number(s): 14.60.St, 11.30.Fs, 13.10.+q, 13.35.Hb

I. INTRODUCTION

The bounds on the anomalous magnetic moment of the τ lepton, a_τ, are much weaker than the ones for the electron and muon. Because of the lepton's short lifetime of $(291.0\pm 1.5)\times 10^{-15}$ sec, its anomalous magnetic moment cannot be measured by a spin precession method and no direct measurement of a_τ exists so far. Because of the impossibility of measuring a_τ by the spin precession method, the present bounds have been obtained by analysis of collision experiments. In that sense, interesting articles, which obtain bounds on a_τ, have been published recently. The OPAL Collaboration [1] uses a reaction proposed for Mendez and Grifolds [2] some years ago. They obtained limits on a_τ from the nonobservation of anomalous $\tau\overline{\tau}\gamma$ production at the CERN e^+e^- collider LEP. In another article, Gonzales-Sprinberg, Santamaria, and Vidal using LEP1, SLAC Large Detector (SLD), and LEP2 data, for tau lepton production, and data from the collider detector at Fermilab (CDF, DØ) and LEP2, for W decays into a tau lepton, established model independent limits on the nonstandard electromagnetic and weak magnetic moments of the tau lepton [3]. In this case the obtained electromagnetic bound is $|a_\tau|<0.009$ at 95% C.L., which is the best bound obtained today but still far away of the theoretical precision for $a_\tau:(1177.3\pm 0.3)\times 10^{-6}$. This weakness of the a_τ bounds is unfortunate since large deviations from the standard model (SM) values are expected for the τ lepton. In particular, in composite models one would expect larger effects for the tau lepton than for the rest of the leptons. In this article we are interested in studying the capability of a $\gamma\gamma$ collider to improve the existing bounds on a_τ. In this respect we study the $\gamma\gamma\to\tau\overline{\tau}$ process which has been studied in the past by different authors [4]. It is an extremely clean process because it has no interference with weak interactions, being a purely QED process. Moreover, the high center of mass energies proposed for this collider make it an adequate laboratory to see anomalous magnetic moment effects which grow with the energy. In this work, we analyze a different set of observables than the ones previously studied by other authors [4]. In particular, we analyze the angular distribution for the universality ratio and asymmetries in the production of polarized τ when the initial photons are polarized too. As far as we know this observable set has not been analyzed yet.

Following Ref. [5], in order to analyze tau magnetic moments, we will use an effective Lagrangian description. Thus, in Sec. II, we describe the effective Lagrangian formalism. In Sec. III, we summarize the helicity formalism, which is used for calculating the cross section. In the same section we present the different used observables. Finally, in Sec. IV we give our conclusions.

II. THE EFFECTIVE LAGRANGIAN APPROACH

In the last few years, effective Lagrangians have been used as an adequate framework to study low energy effects of physics beyond the standard model. Since the SM gives a very good description of all physics at energies available at present accelerators, then one expects that any deviation of the SM can be parametrized by an effective Lagrangian built with the field and symmetries of the SM. In this condition, the effective Lagrangian contains a renormalizable piece, the SM theory and nonrenormalizable operators of dimensions higher than 4 which are suppressed by inverse powers of the high energy physics scale Λ. The leading nonstandard effects will come from the operators with the lowest dimension. Those are dimension six operators. In particular, there are only two six-dimension operators which contribute to the anomalous magnetic moments [6]

$$\mathcal{O}_B=\frac{g'}{2\Lambda^2}\overline{L_L}\phi\sigma_{\mu\nu}\tau_R B^{\mu\nu},$$

$$\mathcal{O}_W=\frac{g}{2\Lambda^2}\overline{L_L}\vec{\tau}\phi\sigma_{\mu\nu}\tau_R\vec{W}^{\mu\nu}, \tag{1}$$

where $L_L=(\nu_L,\tau_L)$ is the tau leptonic doublet and ϕ is the Higgs doublet. $B^{\mu\nu}$ and $W^{\mu\nu}$ are the $U(1)_Y$ and $SU(2)_L$ field strength tensor, and g' and g are the corresponding gauge couplings. Thus, we write our effective Lagrangian as

$$\mathcal{L}_{\mathrm{eff}}=\mathcal{L}_{\mathrm{SM}}+\alpha_B\mathcal{O}_B+\alpha_W\mathcal{O}_W+\mathrm{H.c.} \tag{2}$$

As we are not interested in studying CP violation effects, then we will take the coupling α_B and α_W real. Moreover, we will consider them as free parameters without any further assumption.

The interaction Lagrangian can be written in terms of the physical fields A_μ, Z_μ, and $W^\pm_\mu$. In our particular case, we are only interested in the effective electromagnetic interaction, since we are studying a process which only involves electromagnetic interactions. In this condition the relevant Lagrangian is

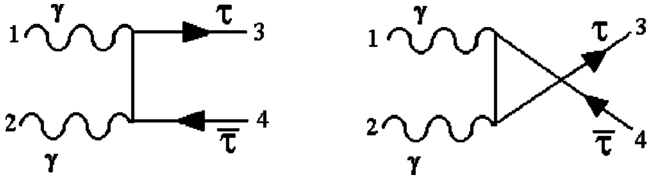

FIG. 1. Feynman graph contributing to the amplitude of the $\gamma\gamma\to\tau\overline{\tau}$ process.

$$\mathcal{L}_{\rm eff}=\mathcal{L}_{\rm SM}+a_{\tau}\frac{e}{4m_{\tau}}\overline{\tau}\sigma_{\mu\nu}\tau F^{\mu\nu}+\dots, \tag{3}$$

where the dots represent nonrelevant pieces of the Lagrangian and $F_{\mu\nu}$ is the electromagnetic field strength tensor. We have expressed the coupling in function of a_τ because it is directly related to the experimental measurement and theoretical calculations. The a_τ constant can be expressed in terms of α_B, α_W, and Λ as follows:

$$a_{\tau}=\frac{\sqrt{2}v m_{\tau}}{\Lambda^2}(\alpha_B-\alpha_W). \tag{4}$$

III. THE $\gamma\gamma\to\tau\overline{\tau}$ PROCESS

In this section we study the $\gamma\gamma\to\tau\overline{\tau}$ process which only involves electromagnetic interactions plus additional magnetic moment couplings given by Eq. (3). This process is of interest for a number of reasons. Increased cross section for high energy and the absence of weak contributions are further complementary features of the two-photon process. In addition, very hard photons at high luminosity may be produced in Compton backscattering of laser light of high energy e^+e^- beams.

The corresponding Feynman diagrams are shown in Fig. 1 and the corresponding amplitude for the process can be written as

$$\mathcal{M}=-ie^2[P_{\tau}(p_1-k_1)T_1+P_{\tau}(p_1-k_2)T_2], \tag{5}$$

where $P_\tau(k)=1/(k^2-m_\tau^2)$ and

$$T_1=\overline{u}(p_1)\left(\gamma_\mu-i\frac{a_\tau}{2m_\tau}\sigma_{\mu\delta}k_1^\delta\right)(\not p_1-\not k_1)\times\left(\gamma_\nu-i\frac{a_\tau}{2m_\tau}\sigma_{\nu\rho}k_2^\rho\right)v(p_2)\epsilon_1^\mu\epsilon_2^\nu,$$

$$T_2=\overline{u}(p_1)\left(\gamma_\nu-i\frac{a_\tau}{2m_\tau}\sigma_{\nu\rho}k_2^\rho\right)(\not p_1-\not k_2)\times\left(\gamma_\mu-i\frac{a_\tau}{2m_\tau}\sigma_{\mu\delta}k_1^\delta\right)v(p_2)\epsilon_1^\mu\epsilon_2^\nu. \tag{6}$$

Bounds on the anomalous coupling a_τ can be obtained from a test of universality in $\gamma\gamma$ colliders by assuming that only the tau lepton has anomalous magnetic moment (muon and electron magnetic moments have been measured quite precisely [7]).

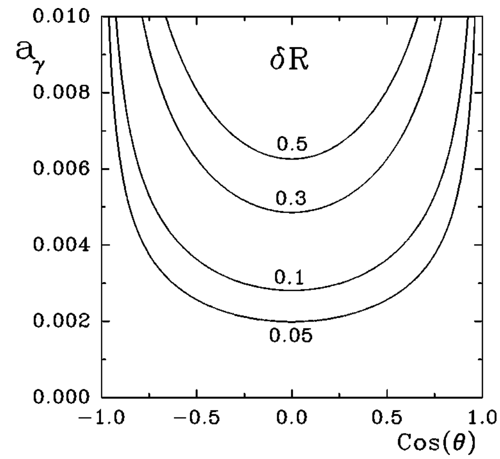

FIG. 2. Contour plot for δR in the $(\cos\theta, a_\tau)$ plane for $\sqrt{s}=200$ GeV.

In order to compare with the experimental data, we define the *angular dependent universality ratio* for unpolarized initial photon and final lepton

$$R(\theta)=\frac{d\sigma/d\Omega|_{(\gamma\gamma\to\tau\overline{\tau})}}{d\sigma/d\Omega|_{(\gamma\gamma\to\mu\overline{\mu})}}. \tag{7}$$

This ratio is a function of the dispersion angle and the tau lepton anomalous magnetic moment a_τ. We have considered final lepton as massless. In this condition the above ratio can be written as

$$R(\theta)\simeq 1+\delta R(\theta,a_\tau), \tag{8}$$

where

$$\delta R(\theta,a_\tau)=4c_\tau^2 s\frac{\sin^2(\theta)}{1+\cos^2(\theta)}[4+c_\tau^2 s\sin^2(\theta)]. \tag{9}$$

For simplicity in the calculation we have used the related constant $c_\tau=a_\tau/(4m_\tau)$. We illustrate the behavior of δR as a function of a_τ and θ, showing in Fig. 2 the contour plot for different values of δR in the $(\cos\theta, a_\tau)$ plane for $\sqrt{s}=200$ GeV. As it is shown in this figure the maximal sensibility of this observable is reached for θ closed to $\pi/2$.

Another way to present the same results is shown in Fig. 3, where we plot R as a function of $\cos\theta$ for different values of a_τ, for $\sqrt{s}=200$ GeV. If the tau lepton is a sequential lepton with the same properties of the electron and muon, then we expect an $R=1$ value. On the other hand, if effects of new physics were relevant for the tau lepton, then we would expect a different value of R. For example, we consider a measurement of R compatible with one (the SM value) within an accuracy of 10%; then we could obtain an a_τ bound of $a_\tau\lesssim 0.003$ for $\sqrt{s}=200$ GeV.

The Photon Linear Collider may be the best alternative to the electron-positron collider. In this collider we have the opportunity to control the initial photon polarization by the inverse Compton scattering of the polarized laser by the electron-positron beam at NLC [8,9]. Adjusting the laser polarization, we can get highly polarized photons. We discuss

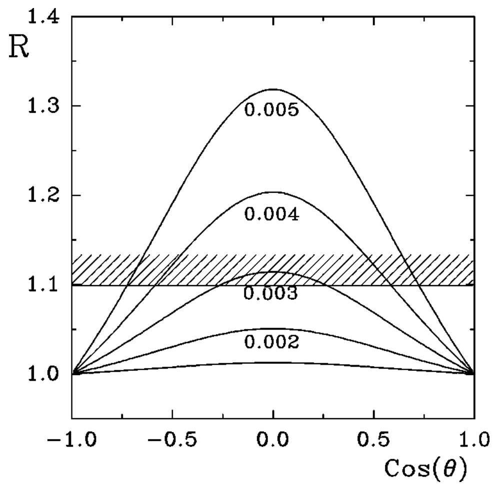

FIG. 3. Universality ratio for $\sqrt{s}=200$ GeV and for different values of a_τ. We have included the limit for a hypothetical measurement of R with an accuracy of 10%.

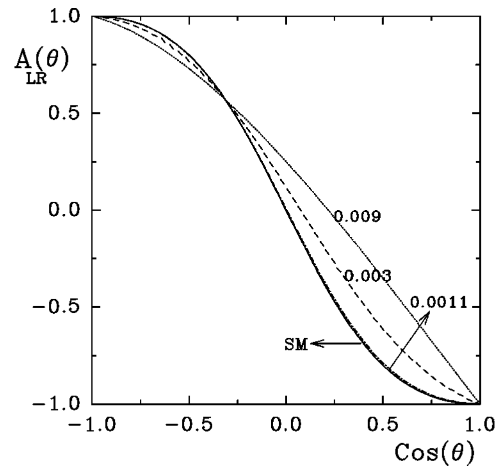

FIG. 4. The angular dependent asymmetry $\mathcal{A}_{LR}(\theta)$ as a function of cos θ for $\sqrt{s}=200$ GeV and for different values of a_τ. We include the curves corresponding to the best bound for a_τ and for the theoretical estimation.

only the case of circular polarized beam. In this condition, we study the process $\gamma\gamma\to\tau\bar{\tau}$ for the polarized tau lepton pair production from circular polarized photon collision ($\gamma_{L,R}\gamma_{L,R}\to\tau_{L,R}\overline{\tau_{L,R}}$) in the center-of-mass (c.m.) frame at tree level in perturbation theory. Due to the high c.m. energies involved, we can consider the final lepton as massless. The polarization of the final tau (anti-tau) can be studied by their decay products. They are referred to as the spin analyzer of the tau lepton. Effectively, the spin polarization of the produced tau is reflected in the distorted distribution of the decay products. Therefore, the τ polarization can be determined from a measurement of the spectrum of the final charged particle in the following decay channels: $\tau^-\to\nu_\tau\pi^-,\nu_\tau\rho^-,\nu_\tau a_1^-,\nu_\tau e^-\overline{\nu_e},\nu_\tau\mu^-\overline{\nu_\mu}$.

In order to simplify the calculation of the cross section when the initial photons and the final tau pairs are polarized we use the helicity amplitude method (HAM). In this section we summarize the principal features of this method.

In order to calculate these amplitudes we follow the rules from helicity formalism and use identities of the type

$$\{\overline{u_\lambda}(p_1)\gamma^\mu u_\lambda(p_2)\}\gamma_\mu=2u_\lambda(p_2)\overline{u_\lambda}(p_1)+2u_{-\lambda}(p_1)\overline{u_{-\lambda}}(p_2), \qquad (10)$$

which is in fact the so-called Chisholm identity, and

$$\not{p}=u_\lambda(p)\overline{u_\lambda}(p)+u_{-\lambda}(p)\overline{u_{-\lambda}}(p), \qquad (11)$$

defined as a sum of the two projections $u_\lambda(p)\overline{u_\lambda}(p)$ and $u_{-\lambda}(p)\overline{u_{-\lambda}}(p)$.

The spinor products are given by

$$s(p_i,p_j)\equiv\overline{u_+}(p_i)u_-(p_j)=-s(p_j,p_i),$$
$$t(p_i,p_j)\equiv\overline{u_-}(p_i)u_+(p_j)=[s(p_j,p_i)]^*. \qquad (12)$$

Using the above rules, which are proved in Ref. [10], we can reduce many amplitudes to expressions involving only spinor products.

For the polarization of the initial photon we take [10] $\epsilon_\lambda^\mu(k)=N\overline{u_\lambda}(k)\gamma^\mu u_\lambda(p)$, where $N=1/\sqrt{4k\cdot p}$ and p^μ is any lightlike vector not collinear to k^μ. We take for p^μ one of the other momenta occurring in the problem. In particular we take $p=p_1$, where p_1 is the 4-momentum of the tau lepton.

For simplicity in the expressions and in the numerical calculation we assign a number for each 4-momentum as is shown in Fig. 1. In this condition, we represent the products $s(p_i,p_j)$ and $t(p_i,p_j)$ with the symbols s_{ij} and t_{ij}, respectively. The corresponding amplitude are written as functions of these symbols:

$$T_1(-1,-1,-1,-1)=-4f_1f_2s_{13}s_{34}t_{12}t_{13},$$

$$T_1(-1,-1,-1,\ 1)=4f_1f_2s_{13}s_{24}t_{13}^2(-1+4c_\tau^2s_{23}t_{23}),$$

$$T_1(-1,\ 1,-1,-1)=8f_1f_2c_\tau s_{13}s_{23}t_{12}t_{13}t_{24},$$

$$T_1(-1,\ 1,-1,1)=-8f_1f_2c_\tau s_{13}s_{23}t_{13}^2t_{34},$$

$$T_1(\ 1,-1,\ 1,-1)=-8f_1f_2c_\tau s_{13}^2s_{34}t_{13}t_{23},$$

$$T_1(\ 1,-1,\ 1,\ 1)=8f_1f_2c_\tau s_{12}s_{13}s_{24}t_{13}t_{23},$$

$$T_1(\ 1,\ 1,\ 1,-1)=4f_1f_2s_{13}^2t_{13}(-1+4c_\tau^2s_{23}t_{23})t_{24},$$

$$T_1(\ 1,\ 1,\ 1,\ 1)=-4f_1f_2s_{12}s_{13}t_{13}t_{34},$$

$$T_2(-1,-1,-1,-1)=4f_1f_2s_{23}s_{34}t_{12}t_{23},$$

$$T_2(-1,-1,\ 1,-1)=4f_1f_2s_{14}s_{23}(-1+4c_\tau^2s_{13}t_{13})t_{23}^2,$$

$$T_2(-1,\ 1,-1,-1)=-8f_1f_2c_\tau s_{13}s_{23}t_{12}t_{14}t_{23},$$

$$T_2(-1,\ 1,\ 1,-1)=-8f_1f_2c_\tau s_{13}s_{23}t_{23}^2t_{34},$$

$$T_2(\ 1,-1,-1,\ 1)=-8f_1f_2c_\tau s_{23}^2 s_{34}t_{13}t_{23},$$

$$T_2(\ 1,-1,\ 1,\ 1)=-8f_1f_2c_\tau s_{12}s_{14}s_{23}t_{13}t_{23},$$

$$T_2(\ 1,\ 1,-1,\ 1)=4f_1f_2s_{23}^2(-1+4c_\tau^2 s_{13}t_{13})t_{14}t_{23},$$

$$T_2(\ 1,\ 1,\ 1,\ 1)=4f_1f_2s_{12}s_{23}t_{23}t_{34}, \qquad (13)$$

where $c_\tau=a_\tau/(4m_\tau)$ and the arguments of the T functions correspond to the helicities of τ, $\overline{\tau}$, and the photons, respectively. After the evaluation of the amplitudes for the corresponding diagrams, we obtain the cross sections of the analyzed processes for each point of the phase space and for different helicities of the particles involved in the process.

In this condition we are ready to study different kinds of observables built with the cross section for polarized particles. The photon, which goes ahead in the positive z direction, is considered polarized left, z being the beam axis. The other initial photon, that goes ahead in the negative z direction, is supposed unpolarized. Thus, we define the following observable and we call it asymmetry:

$$\mathcal{A}_{LR}(\theta)=\frac{d\sigma/d\Omega|_L-d\sigma/d\Omega|_R}{d\sigma/d\Omega|_L+d\sigma/d\Omega|_R}, \qquad (14)$$

where $d\sigma/d\Omega|_{L(R)}$ is the differential cross-section production for left (right) tau. The anti-tau is considered unpolarized and the initial state is prepared as we have explained above. From the general expression for the amplitude it is easy to obtain an analytic result for $\mathcal{A}_{LR}(\theta)$:

$$\mathcal{A}_{LR}(\theta)=\frac{-2\cos\theta+4c_\tau^2 s(2-\cos\theta)\sin^2\theta}{1+\cos^2\theta+16c_\tau^2 s(1+c_\tau^2 s\sin^2\theta/4)\sin^2\theta}. \qquad (15)$$

In Figs. 4 and 5 we show $\mathcal{A}_{LR}(\theta)$ for different values of a_τ for $\sqrt{s}=200$ and 500 GeV, respectively. In particular we plot the curve for $a_\tau=0.009$ which corresponds to the best bound obtained from other authors [3], and the curve for $a_\tau=0.001177$ that corresponds to the theoretical estimation for a_τ. As we can see this asymmetry is an observable sensible to effects of magnetic anomalous moment. In particular for $\sqrt{s}=500$ GeV, we can see an important apartment between the standard prediction ($a_\tau=0$) and the best present bound ($a_\tau=0.009$). In this respect we define the relative apartment between the asymmetry and the SM asymmetry ($a_\tau=0$)

$$\delta=\left|\frac{\mathcal{A}_{LR}(\theta)-\mathcal{A}_{LR}^{SM}(\theta)}{2}\right|\times 100, \qquad (16)$$

where the denominator 2 in the above expression corresponds to the variation range of the asymmetry. This apartment δ can be thought of as related to the experimental precision in the $\mathcal{A}_{LR}$ measurement. In Figs. 6 and 7 we plot δ for $\sqrt{s}=200$ GeV and $\sqrt{s}=500$ GeV, respectively. We can see a significant apartment for moderate values of a_τ, which makes of this asymmetry a useful observable to bound magnetic anomalous moment.

For completeness, we define another related observable involving the total cross section:

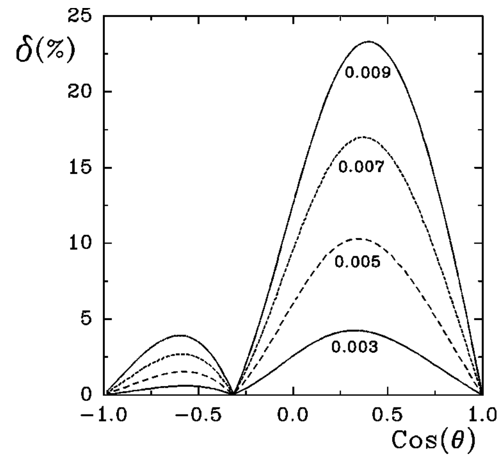

FIG. 6. δ as a function of cos θ for different values of a_τ for $\sqrt{s}=200$ GeV.

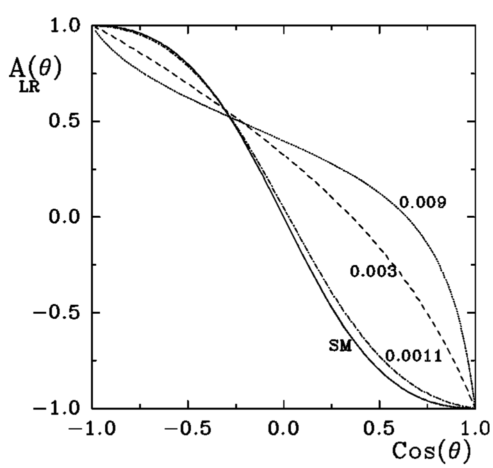

FIG. 5. The same as Fig. 4 but for $\sqrt{s}=500$ GeV.

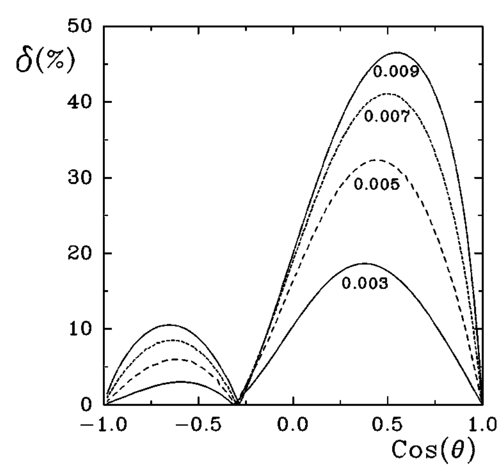

FIG. 7. The same as Fig. 6 but for $\sqrt{s}=500$ GeV.

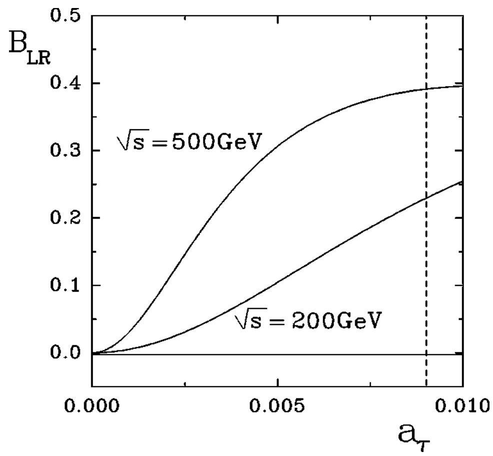

FIG. 8. The integrate asymmetry B_{LR} as a function of a_τ for different values of center-of-mass energies. The vertical line represents the present best bound obtained for a_τ.

$$B_{LR}=\frac{\sigma_L-\sigma_R}{\sigma_L+\sigma_R}, \tag{17}$$

where

$$\sigma_{L(R)}=\int \left.\frac{d\sigma}{d\Omega}\right|_{L(R)} d\Omega. \tag{18}$$

For this observable, it is easy to obtain an analytic expression

$$B_{LR}=\frac{c_\tau^2 s}{1/4+2c_\tau^2 s+2/5c_\tau^4 s^2}. \tag{19}$$

Note that this observable vanishes in the standard model ($a_\tau=0$). In Fig. 8 we show it as a function of a_τ for 200 and 500 GeV center-of-mass energies, respectively. We include a vertical line that represents the current best bound for a_τ. As we can see the B_{LR} values show a significant deviation of the SM value ($B_{LR}=0$).

IV. CONCLUSION

In this work we study a set of different observables which have not been investigated by other authors previously. These observables involve universality test and polarization effects. We have found that, by using these observable sets, it could be possible to improve the actual bounds for a_τ.

ACKNOWLEDGMENTS

We thank CONICET (Argentina), Universidad Nacional de Mar del Plata (Argentina) for their financial support.

[1] The OPAL Collaboration, K. Acherstalf *et al.*, Phys. Lett. B **431**, 188 (1988).
[2] J. A. Grifols and A. Mendez, Phys. Lett. B **255**, 611 (1991).
[3] G. A. Gonzalez-Sprinberg, A. Santamaria, and J. Vidal, Nucl. Phys. **B582**, 3 (2000).
[4] A. Aeppli and A. Soni, Phys. Rev. D **46**, 315 (1992); F. Cornet and J. I. Illana, Nucl. Phys. B (Proc. Suppl.) **37**, 193 (1995); F. del Aguila, F. Cornet, and J. I. Illana, Phys. Lett. B **271**, 256 (1991); F. Cornet and J. I. Illana, Phys. Rev. D **53**, 1181 (1996).
[5] R. Escribano and E. Massó, Phys. Lett. B **301**, 419 (1993).
[6] W. Buchmuller and D. Wyller, Nucl. Phys. **B268**, 621 (1986).
[7] Particle Data Group, C. Caso *et al.*, Z. Phys. C **3**, 1 (1998).
[8] I. F. Ginsburg, Nucl. Phys. B (Proc. Suppl.) **82**, 367 (2000).
[9] M. Hori, Y. Kiyo, and T. Nasuno, Phys. Rev. D **58**, 014005 (1998).
[10] R. Kleiss and W. J. Stirling, Nucl. Phys. **B262**, 235 (1985).